Tecniche di controllo robusto $l_1$ e $l_\infty$
per la regolazione del minimo nei motori a scoppio

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Outline

• Idle speed control problem
• Port-Injection engines description
• Proposed linear control structure
• $l_\infty$ and $l_1$ control via the polynomial equation approach
• Experimentation
• Conclusions
Idle speed control

• It is in charge of handling all situations in which the gas pedal is released. In particular:
  
  – Low regimes during driving, no matter the transmission gear engaged;
  
  – High loads with first gear engaged, e.g. vehicles almost still or slowing moving in steep slopes;
  
  – Engine speed fast dropping from high rpm to the one prescribed at the idle;
  
  – Idle gear and variable loads acting on the crankshaft;

• In all above situations the control problem consists of
  
  – preventing engine stalls
  
  – maintaining the engine speed at the prescribed rpm;
  
  – the rejection of load disturbances;
A port-injection gasoline engine model

- Four interacting subsystems are of interest:
  - the throttle valve
  - the intake manifold
  - the cylinder
  - the crankshaft
The throttle valve dynamics

\[
\dot{\alpha}_e(t) = \frac{1}{\tau_\alpha} \alpha_e(t) + \frac{1}{\tau_\alpha} \alpha(t - d_\alpha)
\]

where:

- \( \alpha \) denotes the throttle valve command (gas pedal)
- \( \alpha_e \) denotes the throttle valve angle
- \( d_\alpha = 20 \text{ ms} \) denotes the electrical actuator delay
- \( \tau_\alpha = 50 \text{ ms} \) mechanic time constant
The intake manifold dynamics

\[ \dot{p}(t) = K_{gas}(F_{th}(\alpha_e(t), p(t)) - F_{cyl}(n(t), p(t))) \]
\[ \dot{q}_a(t) = F_{cyl}(n(t), p(t)) \]

where:

- \( F_{th}(\alpha_e, p) \) is the input air-flow rate. It is a highly nonlinear static function, approximated by a piece-wise linear function of \( \alpha_e \) and \( p \).
- \( F_{cyl}(p, n) \) is the output air-flow rate. It is a highly nonlinear static function, approximated by a piece-wise linear function of \( p \) and \( n \).
- \( K_{gas} \) is the gas constant

The \textit{intake manifold} dynamic is described in terms of the manifold pressure \( p \) and of the amount of air in the cylinder \( q_a \) as follows:
The cylinder dynamics

The cylinder subsystem describes how the torque is generated from fuel combustion. A static map of the form

$$T_{eng} = T_{eng}(q_a, q_b, n, \beta)$$

is usually achieved experimentally where

- $q_a$ and $q_b$ are the total masses of injected fuel and air;
- $n$ the engine speed and $\beta$ the spark advance;

A more convenient way to express the above map at the stoichiometric ratio $\lambda = \frac{q_a}{q_b} \approx 14.66$ (for gasoline) is

$$T_{eng} = T_{pot}(q_a, n)\eta(\beta)$$

where $T_{pot}$ is the maximum potential torque and $\eta(\beta)$ the spark advance efficiency
The crankshaft dynamics

The crankshaft block describes the evolution of the crankshaft revolution speed $n$, whose acceleration depends on the difference between the engine torque $T_{\text{eng}}$ and the load torque $T_{\text{load}}$:

$$\dot{n}(t) = K_j (T_{\text{eng}}(t) - T_{\text{load}}(t))$$

The load torque $T_{\text{load}}(t)$ consists essentially of three distinct amounts:

- Pumping torque
- Friction torque
- Additional torque, due to the auxiliary subsystems powered by the engine (e.g. electrical generator, air conditioner, etc.)
Spark ignition engine cycle

- The dead center events of a four-stroke engine\(^1\) occur when the pistons reach either the top or bottom positions. We denote by \(t_k\) the sequence of times at which they occur.

- Then, the amount of air \(q_a\) loaded by a cylinder during each intake stroke is obtained by integrating the input air-flow \(F_{cyl}\) between two dead centers, i.e.

\[
q_a(t_{k-1}) = \int_{t_{k-2}}^{t_{k-1}} F_{cyl}(n(t), p(t)) dt
\]

- We assume that \(q_a(t) = q_a(t_{k-1})\), \(\forall t \in [t_{k-1}, t_{k+1}]\) is constant during subsequent compression and expansion strokes.

\(^1\)Intake, compression, expansion and exhaust strokes.
An averaged modelling approach

The torque generated during the expansion stroke is averaged along expansion stokes
In the hybrid model the produced torque \( T_{\text{eng}}(t) \) is modelled as a piecewise-constant signal, synchronized with the dead center events.

\[
T_{\text{eng}}(t) = T_{\text{eng}}(t_k) = T_{\text{pot}}(q_a(t_{k-1}), n(t_k))\eta(\beta(t_{k-1})), \quad t \in [t_k, t_{k+1})
\]

where:

- \( q_a(t_{k-1}) \) is the total mass of injected air at the end of the intake stroke. Of course \( q_b(t_{k-1}) = \lambda q_a(t_{k-1}) \);
- \( n(t_k) \) is the value of the engine speed at the beginning of the stroke \( t_k \);
- \( \beta(t_{k-1}) \) is the spark advance for the expansion stroke \( t_k \) decided at time \( t_{k-1} \)
Discrete-time multirate system

- The throttle valve and intake manifold dynamics are discretized at the fast and constant sampling rate $t_f = 12$ ms. The throttle valve commands are provided at even faster sampling rates (4 ms).

- All other dynamics are discretized at every engine stroke (in four-cylinders engines) at variable TDC sampling rates. This correspond to the sampling rate of $t_k = 44$ ms at the speed of 680 rpm. The spark advance commands are also provided at TDC sampling rates.

- A multirate discrete-time LTI plant description is enough for control synthesis purposes because mostly of the nonlinearities can be inverted. The TDC discretized system describes all relevant quantities at dead-center times.

- The model is built up at the nominal idle speed of 680 rpm. Variability in $t_k$ are taken into account but this is not a serious problem for the idle speed control.
Discrete-time multirate control structure

- The **Spark Advance** and **Throttle Valve** SISO controllers have been synthesized on the basis of the following multirate LTI-TD plant description

\[
n(t_k) = \frac{B_1(d)}{A_1(d)} T_{ec}(t_k) + \frac{C_1(d)}{A_1(d)} T_{load}(t_k), \quad T_{ec} \leq T_{pc}
\]

\[
T_{pe}(t_f) = \frac{B_2(d)}{A_2(d)} T_{pc}(t_f)
\]

where

- \( T_{ec} \) is the required produced torque;
- \( T_{pc} \) is the required potential torque;
- \( T_{load} \) is the total load torque;
- \( T_{pe} \) is an estimate of the actual potential torque;
The **SA** controller is in charge to regulate the engine speed. Its main goal is fast rejection of step disturbances $T_{load}$. To reduce consumption, low activity to the command $T_{ec}$ is required.

The **TV** controller is in charge to regulate the dynamic of the required potential torque $T_{pr}$, to be considered as an instantaneous torque reserve for fast compensation of load disturbance $T_{load}$. Its main goal is to provide a good tracking of $T_{pr}$ by $T_{pe}$.

The **reference actuator** block is in charge to translate the $T_{ec}$ and $T_{pe}$ requirements in terms of spark advance $\beta$ and throttle valve angle $\alpha$. Moreover, all nonlinearities are here inverted.
Spark advance controller design

- Fast rejection of piecewise constant load disturbances
- Fuel consumption minimization during transients
- Good tracking performance on the engine speed
- Industrial practice typically makes use of PID-like or other non-model based control design techniques

- $l_\infty$ and $l_1$ finite-dimensional optimal control
A polynomial equation approach

Assuming for simplicity \( r(k) = 0 \)

\[
Y(d) = \frac{B(d)}{A(d)} U(d) + \frac{C(d)}{A(d)} D(d)
\]

- \( d \) is the one-step delay,
- \( U(d), Y(d) \) and \( D(d) \) \( \mathcal{D} \)-transforms of input, output and disturbance,
- \( \frac{B(d)}{A(d)} \) strictly causal and \( \frac{C(d)}{A(d)} \) causal

Assume that the disturbance sequence \( d(t) \) is a polynomially unbounded sequence with rational \( \mathcal{D} \)-transform

\[
D(d) := \frac{B_d(d)}{A_d(d)}
\]

with roots of \( A_d(d) \) in \(|d| \geq 1\).

Assume also:

\[
(A.1) \begin{cases}
(A, B) \text{ coprime with } A(0) \neq 0, B(0) = 0 \\
(A_d, B_d) \text{ coprime with } A_d(0) \neq 0.
\end{cases}
\]
A polynomial equation approach

Define the feedback action between the output $y(t)$ and $u(t)$ as

$$U(d) = -\mathcal{K}(d)Y(d)$$

with

$$\mathcal{K}(d) = \frac{S(d) + A(d)Q(d)}{R(d) - B(d)Q(d)}$$

with the polynomial pair $(R, S)$ satisfying

$$A(d)R(d) + B(d)S(d) = 1$$

and the free Youla transfer function $Q$ causal and asymptotically stable.

Perform the following causal/anticausal decompositions

$$B = B^- B^+, \quad A_d = A_d^- A_d^+, \quad B_d = B_d^- B_d^+, \quad C = C^- C^+$$

where

- $B^+$ is stable, viz. free of roots in $|d| \leq 1$ and
- $B^-$ is monic unstable, viz. with all of its roots in $|d| \leq 1$
Deadbeat ripple-free parameterization

Assume

\begin{align*}
(A_d, B) \text{ coprime polynomial pair} \\
A_d \text{ factor of } (1 - d)C^-, \text{ i.e.} \\
(1 - d)C^- = GA_d, \text{ for some polynomial } G.
\end{align*}

The first assumption is required to ensure both the dead-beat and ripple-free properties, whereas the second needs only if ripple-free responses are of interest.

**Proposition** - Let (A.1)-(A.2) be fulfilled. Then, the Youla parameter \( Q \) yielding all ripple-free dead-beat controllers and the corresponding closed-loop responses \( Y(d) \) and \( \Delta U(d) = (1 - d)U(d) \) can be parameterized in terms of an arbitrary polynomial \( W(d) \) as follows

\begin{align*}
Q &= \frac{Z_o + A_d(T_o + B^+W)}{C^+B^+B_d^+} \quad (1) \\
Y &= Y_o - C^-B^-B_d^-[T_o + B^+W] \quad (2) \\
\Delta U &= GB_d^- \left(SC^+B_d^+ + A[V_o + A_dW]\right) \quad (3)
\end{align*}

with \( G \) as in (A.2)

where \((Y_o, Z_o)\) is the unique m.d. solution w.r.t. with \( Y \) (i.e. \( \deg Y < \deg C^-B^-B_d^- \)) of

\[ ZC^-B^-B_d^- - A_dY = CB_dR \]

while \((V_o, T_o)\) is the unique m.d. solution w.r.t. \( T \) (i.e. \( \deg T_o < \deg B^+ \)) of

\[ -A_dT + B^+V = Z_o \]
Design objectives - Performance

Observe that the degrees of both \( Y(d) \) and \( \Delta U(d) \) grow monotonically with the degree \( w \) of \( W(d) \). Thus, \( w \) is a control design parameter.

Fast disturbance rejection

(P.1) \( \min_{W \in \mathbb{R}^w[d]} \| Y \|_{A_\infty} \)

(P.2) \( \min_{W \in \mathbb{R}^w[d]} \| Y \|_{A_\infty} \) subject to \( \| \Delta U \|_{A_\infty} < \gamma_1 \)

Minimization of the control effort

(P.3) \( \min_{W \in \mathbb{R}^w[d]} \| \Delta U \|_{A_\infty} \)

(P.4) \( \min_{W \in \mathbb{R}^w[d]} \| \Delta U \|_{A_\infty} \) subject to \( \| Y \|_{A_\infty} < \gamma_2 \)

- \( \| H(d) \|_{A_\infty} := \| h_k \|_{\infty} \) where \( H(d) := \sum_{k=0}^{\infty} h_k d^k \)
- For all problems the cost monotonically decreases as \( w \to \infty \)
- All formulations give rise to finite dimensional linear programming problems
Design objectives - Robustness

Under additive unstructured causal LTV perturbations $\Delta P_\gamma$, with $\|\Delta P_\gamma\|_A < \gamma$ one has that

$$\frac{\|Y_\gamma - Y\|_{A\infty}}{\|Y\|_{A\infty}} \leq \frac{\gamma\|M\|_A}{1 - \gamma\|M\|_A}$$
$$\frac{\|U_\gamma - U\|_{A\infty}}{\|U\|_{A\infty}} \leq \frac{\gamma\|M\|_A}{1 - \gamma\|M\|_A}$$

where $M$ is the nominal control sensitivity function and $\|H(d)\|_A = \|h_k\|_1$ with $H(d) := \sum_{k=0}^{\infty} h_k d^k$

Then, the upper-bound on the maximum relative errors can be made as small as possible by minimizing $\|M\|_A$. In fact, $\gamma\|M\|_A \ll 1$ implies

$$\frac{\gamma\|M\|_A}{1 - \gamma\|M\|_A} \approx \gamma\|M\|_A$$

The nominal control sensitivity function

$$M = \frac{U(d)}{B_d/A_d} = \frac{M_1(d) + M_2(d)W(d)}{B_d^+}$$

is a polynomial too provided that either

- $B_d^+$ is a factor of both polynomials $M_1$ and $M_2$
- $B_d^+$ is a scalar
Design objectives - Robustness

Robust designs

(P.5) \( \min_{W \in \mathbb{R}^w[d]} \|M\|_A \)

(P.6) \( \min_{W \in \mathbb{R}^w[d]} \|Y\|_{A\infty} \) subject to \( \|M\|_A < \gamma_3 \)

Robust minimization of the control effort

(P.7) \( \min_{W \in \mathbb{R}^w[d]} \|\Delta U\|_{A\infty} \) subject to \( \|M\|_A < \gamma_4 \)

(P.8) \( \min_{W \in \mathbb{R}^w[d]} \|M\|_A \) subject to \( \|\Delta U\|_{A\infty} < \gamma_5 \)

• For all problems the cost monotonically decreases as \( w \) goes to \( \infty \)

• All formulations give rise to finite dimensional linear programming problems
Experimental results

- The control structure has been implemented on the ECU of a commercial 1.4L Volkswagen Polo engine

- The spark advance controller has been synthesized by minimizing the control effort

\[
\min_{W \in \mathbb{R}^{\omega \times [\nu]}} \| \Delta U \|_{A_\infty}
\]

- Step load disturbances have been considered, viz.

\[
\frac{B_d}{A_d} = \frac{1}{(1 - d)}
\]

- The orders of the SA and TV controllers were 5 and, respectively, 3

- Several tests have been accomplished
  - Response to load step disturbances
  - Transients towards idle
  - Rapid variations of the reference speed
Step disturbance at time $t = 71.5$

- Overshoot on the engine speed halved w.r.t. PID/LQ control
- Believed mainly due to the penalization of $\|\Delta U\|_{A_\infty}$

- Modest fluctuation of the idle speed around the target value
- No saturation on the spark advance efficiency
- Other commands rather smooth
Transients towards idle

- The engine speed drops from high rpm to the idle speed reference value
- Transient is fast, smooth and well dumped
- In this test PID/LQ controllers usually exhibit large undershoots
- Also the commands are reasonable
- A gas pedal stroke at the end the plot causes a fast change of the reference speed
- A fast and smooth transient follows
Fast changes of the reference speed

- Usually a severe test
- Fast reference speed changes caused by gas pedal strokes
- PID/LQ usually produces large undershoots and fluctuations around the nominal idle speed
- Here, on the contrary, the responses are bloody good
- Neither undershoots nor fluctuations are practically observed
- Also the control effort is small
Conclusions

• $l_\infty$ and $l_1$ optimal control techniques have been shown of potential interest for idle speed control problems in automotive industry

• Design techniques based on the polynomial equation approach make this class of controllers easily understandable and solvable with standard mathematical tools

• Also it allows to have some free control design parameters for both modulating the numerical burdens and permitting a fine tuning of the controllers in road tests

• Remarkable improvements w.r.t. to PID/LQ control have been reported by Magneti Marelli Powertrain’s experts

• The control structure has been patented by Magneti Marelli Powertrain and it is actually used in some of its commercial ECUs