Idle Speed Control for GDI Engines using Robust Multirate Hybrid Command Governors

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***Abstract*— The design of an idle speed controller for automotive GDI engines is considered. A hybrid model of a GDI engine operating in stratified mode is presented. The idle speed control problem is formalized as a constrained optimal control problem where fuel consumption has to be minimized. A sub-optimal but effective and easily implementable solution is obtained by resorting to the *Command Governor* methodology for a discrete-time abstraction of the hybrid model. Simulation results of the hybrid closed-loop system are presented.**

I. Introduction

The main targets of the design of 4-stroke gasoline engines for passenger cars are: improvement of safety, driveability and comfort, minimization of fuel consumption and compliance with the emission standards.

High fuel economy, as well as high driving performances, can be achieved by modern Gasoline Direct Injection (GDI) engines, which are equipped with a fuel system that directly injects the gasoline into the cylinders. The advantages of GDI technology are extensively illustrated in [1]. When GDI engines operate in the stratified mode with lean mixtures (i.e. high air/fuel ratio), fuel consumption can be reduced by to 20-25% at low loads and low engine speed.

In a typical driving cycle, the most significant reduction of fuel consumption is obtained in the idle speed operation mode (that is when the gear is neutral and the gas pedal is released). In fact, in idle speed control the main objective is the minimization of fuel consumption. The difficulty of the problem lies in the load variations coming from the intermittent use of devices powered by the engine, such as the air conditioning system and the steering wheel servo-mechanism, which may cause engine stall. Interesting results on idle speed control, obtained by applying different approaches, have been presented in [2], [3], [4], [5], [6].

In this work we formalize the idle speed control problem as a robust constrained optimal control problem for a hybrid model of a GDI engine operating in stratified mode.

The adoption of a hybrid formalism allows us to describe the cyclic behavior of the engine, thus capturing the effect of each fuel ignition on the generated torque and the interaction between the discrete torque generation and the continuous power-train and air dynamics. The use of a hybrid modeling framework is particularly interesting in idle speed control since

- the frequency of engine cycles is very low (in fact, it is the minimum value of crankshaft revolution speed at which the engine can operate), and
- in the most critical conditions, an improper control action for a single engine cycle may cause engine stall.

In the idle speed hybrid optimal control problem, the cost functional to be minimized is fuel consumption. The optimization is subject to constraints on engine speed, air-to-fuel ratio, and control inputs. Furthermore, robustness with respect to disturbance and unmodeled dynamics should be guaranteed.

A sub-optimal, but effective and easily implementable, solution to the hybrid optimal control problem has been presented by the authors in [7]. The proposed solution is based on the *Command Governor* methodology[1]. Two switching LQ optimal controllers are employed as a primal control structure for ensuring nominal closed-loop stability and performance (fuel consumption minimization) under linear regimes. The command governor changes the nominal set-point (the engine speed and the air-to-fuel ratio) in order to impose at each time instant the lowest possible engine speed compatible with the fulfillment of all prescribed constraints.

In [7], to cope with the difficulty of handling the complex behavior of the engine hybrid model, the controller has been designed for a discrete-time model that approximates the evolutions of the engine hybrid model. Then, the correct behavior of the controller has been tested by extensive simulations of the closed–loop hybrid model obtained by connecting the model

[1]A command governor is a nonlinear device which is added to a pre-compensated control system. The latter, in the absence of the command governor, is designed so as to perform satisfactorily in the absence of constraint violations. Whenever necessary, the command governor modifies the reference to the closed-loop system so as to avoid violation of constraints. The usage of command governors allows to guarantee constraint satisfaction with small computation efforts.

Fig. 1. GDI engine hybrid model.

of discrete–time idle speed controller to the GDI engine hybrid model.

In this work we propose a similar approach to the controller design, with the same controller structure that employs two LQ controllers and a command governor. However, by using relaxation and robust techniques, we obtain a feedback controller that is *correct by design,* in the sense that it ensures constraint satisfaction for the hybrid model of the GDI engine. This result is of particular interest in automotive applications where usually controllers are designed for continuous time mean–value models of the plant (that do not represent the discrete phenomena due to the four–stroke engine cycle) and specification satisfaction is always assessed by simulation results only. Hybrid engine and power–train controllers with guaranteed properties have been investigated by two of the authors in the past. In particular, in [8], [9] a hybrid controller the addresses the drivability problem of the cut–off control was proposed using the relaxation technique adopted in the present work.

II. GDI ENGINE HYBRID MODEL AND PROBLEM FORMULATION

In this section a nonlinear hybrid model of a 4–stroke 4–cylinder GDI engine is presented. The proposed model has been developed and identified in collaboration with Magneti Marelli Powertrain (Italy) on the basis of the experimental data obtained from a 2–liter GDI engine. Extensive simulations of the engine hybrid model have been performed in *Matlab/Simulink*^{TM}.

Since in this work we consider the design of an idle speed controller for GDI engines operating in stratified charge, the description of the hybrid model for the homogenous charge is omitted[2]. When GDI engines are controlled in stratified charge, the control inputs are:

- the command to the throttle valve, referred to as α – which is used to control the amount of air loaded by the cylinders during the intake stroke;
- the mass of fuel injected in each engine cycle, referred to as q_b.

In fact, in the stratified charge, spark ignition must be synchronized with fuel injection and cannot be used as a control input.

[2]Switching between stratified and homogenous charges will be investigated in future works.

Fig. 2. Hybrid automata model of the cylinders.

As described in Figure 1, the GDI engine hybrid model is composed of four interacting subsystems: the *throttle valve*, the *intake manifold*, the *cylinders* and the *crankshaft*.

The dynamic of the *throttle valve* is modeled by a first-order lag filter with input delay:

$$\dot{\alpha_e}(t) = \frac{1}{\tau_\alpha}\alpha_e(t) + \frac{1}{\tau_\alpha}\alpha(t - d_\alpha) + \delta_\alpha(t) \qquad (1)$$

where: α and α_e denote, respectively, the throttle command and the throttle angle; d_α models the actuator delay; $\delta_\alpha(t)$, with $|\delta_\alpha(t)| \leq \Delta_\alpha$, represents bounded model uncertainties.

The *intake manifold* dynamics is described in terms of the manifold pressure p as follows:

$$\dot{p}(t) = K_{gas}[F_{th}(\alpha_e(t)) - F_{cyl}(p(t), n(t))] + \delta_p(t) \qquad (2)$$

$$q_a(t) = \frac{k_1}{n}F_{cyl}(p(t), n(t)) + \delta_q(t) \qquad (3)$$

According to (2), the evolution of the manifold pressure p depends on the difference between the manifold input air–flow rate passing through the throttle valve F_{th} and output air–flow rate F_{cyl} (the latter being a function of the manifold pressure p itself and the crankshaft speed n).

We denote by t_k^{dc}, with $k = 1, \ldots, \infty$, the sequence of time instants at which the pistons reach a dead center, i.e. either the lower most (bottom dead center) or upper most (top dead center) positions. The output equation (3) evaluated at time $t = t_k^{dc}$ gives the amount of air mass $q_a(k) = q_a(t_k^{dc})$ loaded by the cylinder that concluded the intake stroke at time t_k^{dc}. The bounded disturbances $|\delta_p(t)| \leq \Delta_p$ and $|\delta_q(t)| \leq \Delta_q$ model plant uncertainties.

The *cylinders* block describes the torque generation mechanism of the internal combustion engine. Four cylinder engines have behaviors significantly simpler than other engines, due to the fact that at each time each cylinder is in a different stroke of the engine cycle: either in intake, compression, expansion or exhaust. The *cylinders* subsystem is modeled by the hybrid automaton depicted in Figure 2. In this model the end of a stroke and the beginning of the subsequent one is represented by the dead–center self–loop transition that is executed when the crankshaft angle θ reaches 180 degrees. This transition defines the dead–center time sequence t_k^{dc}.

The torque $\mathbf{T}_{eng}(t)$ generated by the engine is a piece–wise constant signal, with discontinuity points at times t_k^{dc},

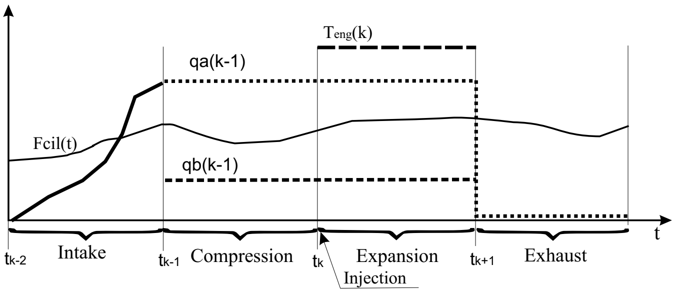

Fig. 3. Evolution of the intake air–flow F_{cyl}, loaded air q_a, injected fuel q_b, generated torque $\mathbf{T}_{eng}(t)$.

synchronized with the dead center events. During the k–th expansion stroke, the amount of the engine torque depends in a nonlinear fashion on: the mass of injected fuel $q_b(k)$, the mass of loaded air $q_a(t^{dc}_{k-1})$, and the value of the engine speed at the beginning of the stroke $n(t^{dc}_k)$. It is customary to express the engine torque in terms of the normalized air–to–fuel ratio of the mixture $\lambda(k)$ during the k-th expansion stroke, which is defined as

$$\lambda(t) = \lambda(k) = \frac{q_a(t^{dc}_{k-1})}{q_b(k)} \left(\frac{q_{a0}}{q_{b0}}\right)^{-1} \quad (4)$$

for $t \in [t_k, t_{k+1})$, where $\frac{q_{a0}}{q_{b0}}$ stands for the stoichiometric air–to–fuel ratio. Engine torque is then expressed as

$$\mathbf{T}_{eng}(t) = \mathbf{T}_{eng}(k) = \mathbf{T}_{eng}(q_b(k), \lambda(k), n(t_k)) \quad (5)$$

for $t \in [t_k, t_{k+1})$.

Notice that both the generated torque $\mathbf{T}_{eng}(k)$ and the normalized air–to–fuel ratio $\lambda(k)$ depend on the amount of air $q_a(t^{dc}_{k-1})$ loaded in the cylinder during the previous intake stroke: the one step delay models the compression stroke which is located between intake and expansion. The timing of the engine internal variables is depicted in Figure 3.
Finally, the *crankshaft* block describes the evolution of the crankshaft revolution speed n (in rpm) and the crankshaft angular position θ (in degrees),

$$\begin{aligned} \dot{n}(t) &= K_J(\mathbf{T}_{eng}(t) - \mathbf{T}_{load}(t)) && (6)\\ \dot{\theta}(t) &= 6\ n(t) && (7) \end{aligned}$$

In (6), $\mathbf{T}_{load}$ represents the load torque, which consists of the sum three distinct amounts: the pumping torque $\mathbf{T}_p$ (a known constant), the friction torque $\mathbf{T}_{fr}$ (linearly depending on n and known), and the torque $\mathbf{T}_d$ due to the auxiliary subsystems powered by the engine[3]. The idle speed controller has to guarantee the requested performances robustly with respect to the action of the torque $\mathbf{T}_d$, which is seen as a disturbance affecting the evolution of the engine speed. Then, to address the robustness requirement, it is convenient to split the torque $\mathbf{T}_d$ as the sum of a *predictable* torque (whose transition to nozero values will be known in advance) and an *unpredictable* torque $\mathbf{T}_{unp}$ (collecting bounded unmeasurable disturbances and model uncertainties):

$$\mathbf{T}_d(t) = \mathbf{T}_{pr}(t) + \mathbf{T}_{unp}(t)$$

where

$$\mathbf{T}_{unp}(t) \in \mathbf{D}_1, \qquad \mathbf{T}_{pr}(t) \in \mathbf{D}_2, \qquad \forall t.$$

As an example, the air conditioning subsystem generates a load which can be considered predictable. In fact, we can assume to know, some time in advance, both the time of the air conditioning switching and the corresponding value of load. This information can be exploited in order to achieve less conservative results.

A. Problem formulation

The idle speed control problem is formalized as a robust constrained optimization control problem. The cost function to minimize is fuel consumption. Constraint variables are:

- the engine speed, to be kept within prescribed operative constraints to prevent engine stall and limit fuel consumption;
- the normalized air-to-fuel ratio, which is subject to constraints due to tail–pipe emission specifications;
- fuel injection and throttle valve command, both subject to amplitude constraints and the latter subjet to a slew–rate constraint also.

Robustness should be achieved with respect to parameter uncertainties and unmodeled dynamics, represented by state disturbances and engine torque load disturbances.

Usually, the specification for the idle speed control is to maintain the engine speed around a nominal reference value n_r. In steady-state fuel consumption is strictly related to the nominal value n_r, in that the lower n_r the lower the fuel consumption. Further, n_r is a fixed reference value that is determined by trading-off between fuel consumption and the need of avoiding the engine to stall during transients due to load disturbances. Then, because fixed, in some situations n_r could be remarkable higher than strictly necessary to keep the engine running. To optimize fuel consumption, we allow the engine speed to vary in an interval around the nominal value.

The specification for idle speed control design is formalized as follows:

$$\begin{aligned}
&\min_{\alpha(t), q_b(k)} \sum_{0}^{\infty} q_b(k) \\
&n(t) \in [710, 790]\ (rpm), \quad \lambda(t^{dc}_k) \in [0.8, 3.5], \\
&q_b(t^{dc}_k) \geq 1\ (mg), \\
&\alpha(t) \geq 0\ (degree), \quad \dot{\alpha}(t) \in [-5, 5]\ (degree/s), \\
&\mathbf{T}_{unp}(t) \in \mathbf{D}_1 = [3, 8]\ (Nm), \\
&\mathbf{T}_{pr}(t) \in \mathbf{D}_2 = [0, 12]\ (Nm), \\
&|\delta_\alpha| \leq \Delta_\alpha = 0.2 (degree/s), \\
&|\delta_p| \leq \Delta_p = 500 (mbar/sec), \\
&|\delta_q| \leq \Delta_q = .8 (mg/sec).
\end{aligned}$$

[3]Such as air conditioning compressor, steering pump, electric generator.

III. Discrete–event based engine relaxed model

The hybrid model presented in Section II is linearized about the operating point corresponding to the nominal engine speed for idle control $n_0 = 750\ (rpm)$ and the disturbance torque $\mathbf{T}_d = \mathbf{T}_{d0}$. The throttle valve is assumed to be controlled by a discrete time feedback with sampling period $T_c = 10$ ms. At nominal engine speed n_0, the time between to subsequent dead centers is 40 ms, i.e. exactly 4 times the throttle control sampling time T_c. We assume that the throttle control sampling is synchronized[4] with dead center events, so that at each dead–center time t_k^{dc} there is a throttle sampling event, while the remaining three occur with a delay of 10, 20 and 30 ms. Let t_h^{th} denote sequence of (not uniform) sampling times of the throttle control and let $\tau_h = t_{h+1}^{th} - t_h^{th}$ represent the sequence of time interval lengths. Assume that dead center events occur on the subsequence $t_h|_{h=4i}$, for $i = 0, 1, \ldots, \infty$. When the engine does not run at the nominal engine speed n_0, the throttle sampling sequence is not uniform and the time intervals $\tau_h|_{h=4i-1}$ before dead centers may vary from $(30000/790-30) \approx 7.97$ ms to $(30000/710-30) \approx 12.25$ ms.

Thus, for any evolution of the hybrid engine model presented in Section II, the state evolution sampled on the time sequence t_h^{th} satisfy the indeterministic discrete dynamics:

$$\begin{aligned} x_p(t_{h+1}^{th}) &= A(\tau_h)x_p(t_h^{th}) + B_1(\tau_h)u_1(t_h^{th}) \\ &\quad + B_2(\tau_h)u_2(t_h^{th}) + B_d(\tau_h)d(t_h^{th}) \quad (8)\\ y(t_h^{th}) &= Cx_p(t_h^{th}) \quad (9) \end{aligned}$$

where, assuming $d_\alpha = 20ms$,

$$x_p(t_h^{th}) = \begin{bmatrix} n(t_h^{th}) - n_0 \\ p(t_h^{th}) - p_0 \\ \alpha_e(t_h^{th}) - \alpha_{e0} \\ q_b(t_{h-1}^{th}) - q_{b0} \\ \alpha(t_h^{th}) - \alpha_0 \\ \alpha(t_{h-1}^{th}) - \alpha_0 \\ \alpha(t_{h-2}^{th}) - \alpha_0 \end{bmatrix}$$

$$d(t_h^{th}) = \begin{bmatrix} \mathbf{T}_d(t_h^{th}) - \mathbf{T}_{d0} + \delta_1(t_h^{th}) \\ \delta_\alpha + \delta_2(t_h^{th}) \\ \delta_p + \delta_3(t_h^{th}) \\ \delta_q + \delta_4(t_h^{th}) \end{bmatrix}$$

$$y(t_h^{th}) = \begin{bmatrix} n(t_h^{th}) - n_0 \\ \lambda(t_h^{th}) - \lambda_0 \end{bmatrix},$$

$\delta_1, \ldots, \delta_4$ are disturbances representing errors due linearization, $u_2(t_h^{th}) = [\alpha(t_h^{th}) - \alpha_0]$, and the control input $u_1(t_h^{th}) = [q_b(t_h^{th}) - q_{b0}]$ is synchronous with the dead center events:

$$u_1(t_h^{th}) = u_1(t_{h+1}^{th}) = u_k(t_{h+2}^{th}) = u_1(t_{h+3}^{th}) \quad (10)$$

for all $h = 4i$ with $i = 0, 1, \ldots, \infty$. Equivalently, the hybrid state sampled on the dead center event sequence t_k^{dc} always satisfy the equation

$$x_p(t_{k+1}^{dc}) = \widehat{A}x_p(t_k^{dc}) + \widehat{B}\widetilde{u}(t_k^{dc}) + \widehat{B_d}\widetilde{d}(t_k^{dc}) + \xi(t_k^{dc}) \quad (11)$$

[4]Notice that this sampling strategy has the advantage that there is no drift between the throttle control and the engine cycle.

where

$$\widetilde{u}(t_k) = \begin{bmatrix} u_1(t_h^{th}) \\ u_2(t_{h+3}^{th}) \\ u_2(t_{h+2}^{th}) \\ u_2(t_{h+1}^{th}) \\ u_2(t_h^{th}) \end{bmatrix}, \qquad \widetilde{d}(t_k) = \begin{bmatrix} d(t_{h+3}^{th}) \\ d(t_{h+2}^{th}) \\ d(t_{h+1}^{th}) \\ d(t_h^{th}) \end{bmatrix}$$

for $h = 4i$ such that $t_h^{th} = t_k^{dc}$; $\xi(t_k^{dc}) \in \mathbf{R}^n$ is a bounded disturbance over–bounding model uncertainties, linearization errors and not uniform dead center events and throttle sampling; and $\widehat{A} = A^4(T_c)$,

$$\begin{aligned} \widehat{B} &= [\ B(T_c) \quad A^3(T_c)B_2(T_c) \quad A^2(T_c)B_2(T_c) \\ &\qquad A(T_c)B_2(T_c) \quad B_2(T_c)\ ] \\ B(T_c) &= [\ A^3(T_c)B_1(T_c) + A^2(T_c)B_1(T_c) \\ &\qquad + A(T_c)B_1(T_c) + B_1(T_c)\ ] \\ \widehat{B_d} &= [\ A^3(T_c)B_d(T_c) \quad A^2(T_c)B_d(T_c) \\ &\qquad A(T_c)B_d(T_c) \quad B_d(T_c)\ ] \end{aligned}$$

We proposed to use of a command governor (CG) for modifying on-line the desired value of engine speed and normalized air–to–fuel ratio, so that the prescribed constraints are never violated, irrespective of all possible load disturbance occurrences, and fuel consumption is optimized. For the engine at hand, it results that fuel consumption is minimized when

$$n_r = 710\ (rpm), \quad \lambda_r = 2$$

Then, the basic strategy underlying the use of a CG will be that to apply the nominal reference values n_r and λ_r and let the CG to modify them on-line whenever their application would lead to constraint violation. For details on CG theory the reader is referred to [7].

IV. Controller synthesis

In this section the design of the proposed idle control is presented. The controller consists of two nested loops:

- a switching LQ controller in the inner loop, whose objective is the minimization of fuel consumption during transients;
- a CG in the outer loop, whose objective is the minimization of fuel consumption during steady states and the verification of the constraints.

The inner loop and the outer loop are, respectively, described in Section IV-A and in Section IV-B below. Simulation results of the closed–loop hybrid system are reported in Section IV-C.

A. Primal Control

The CG approach requires preliminarily the design, if not given, of a primal stabilizing controller which, because is supposedly to be used along with a CG, is designed without tacking into account the prescribed constraint. The one used here is depicted in Fig. 4. In order to have zero tracking error in steady-state we require the use of an integral action. This is done be resorting to the incremental model approach [10] which consists of rewriting the model described in Section III

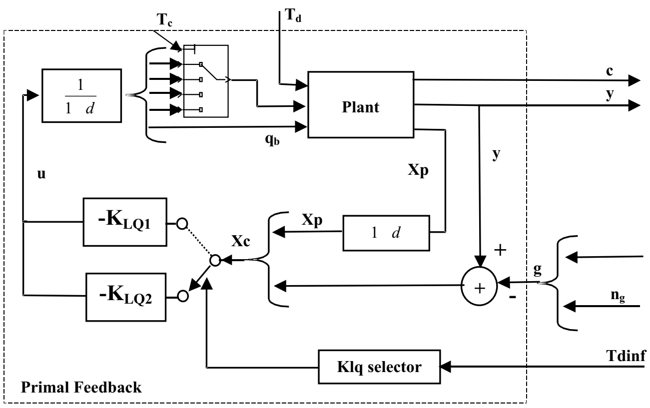

Fig. 4. Primal hybrid feedback control structure.

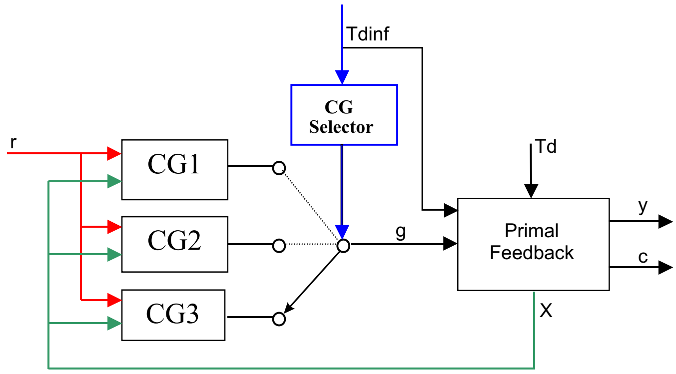

Fig. 5. Hybrid CG structure.

in terms of the extended state $x_c(t)$ and incremental input $\delta u(t) := u(t+1) - u(t)$

$$\begin{aligned} x_c(t+1) &= \Phi x_c(t) + G\delta u(t), \quad x_c(t) := \begin{bmatrix} \delta x_p(t) \\ \varepsilon(t-1) \end{bmatrix} \\ \varepsilon(t) &= H x_c(t) \end{aligned}$$

where $\delta x_p(t) := x_p(t+1) - x_p(t)$ and $\varepsilon(t-1) = y(t-1) - g(t-1)$, $g(t)$ being the reference signal. Then, optimal LQ state feedbacks of the form

$$\delta u(t) = -K_{Lq} x_c(t) \tag{12}$$

which minimizes the following quadratic cost

$$J = \sum_{t=0}^{\infty} \|\varepsilon(t)\|^2_{\Psi_\varepsilon} + \|\delta u(t)\|^2_{\Psi_u} \tag{13}$$

with $\Psi_\varepsilon = \Psi_\varepsilon^{'} \geq 0, \quad \Psi_u = \Psi_u^{'} > 0$ can be easily determined. In particular, we have found convenient to determine two different LQ state feedback control laws, each one well suited to deal with a specific situation. Moreover, a supervisor (K_{lq} Selector) is in charge to identify when each specific controller has to be put in the loop on the basis of the input T_{dinf} that indicates in advance the state of the (ON-OFF) predictable disturbance. Specifically, the two LQ control laws have been designed to work well during the occurrence of the following conditions: 1) “predictable disturbance on/off or off/on transitions” (Lq2) or “no predictable disturbance transitions” (Lq1). The main reason for using two state feedback control laws instead of a single one is that of having different gains during large transient occurrences and steady-state operations. This is convenient for trading-off between fuel consumption minimization and fast transients achievement. In fact, for fuel consumption minimization the weight Ψ_u in the cost has to be chosen remarkably larger than Ψ_ε. Under small disturbances this choice ensures small fuel consumptions and the embedded integral action ensures zero tracking error in steady-state. However, sluggish responses result which cannot be acceptable especially after a large load disturbance change. In such a case, more active control actions are desired. It is worth pointing out that only the fuel consumption due to transients can be optimized by a suitable choice of the control law. On the contrary, the usually predominant amount necessary for supporting the engine during steady-states depends only by constructive details and actual loads which cannot be modified by a specific feedback.

Notice that the overall closed-loop stability can be verified by testing existence of a single symmetric positive definite matrix $P = P' > 0$ which jointly satisfy [11]

$$\Phi_1^{'} P \Phi_1 - P < 0 \text{ and } \Phi_2^{'} P \Phi_2 - P < 0$$

with $\Phi_1 = \Phi + GK_{Lq1}$ and $\Phi_1 = \Phi + GK_{Lq1}$.

B. *CG application (External Loop)*

Accordingly to the above primal control structure, we have designed a bank of three CG’s (see Fig. 5), each one in charge to deal with a different situation. In particular:

CG1 : Lq1 in the loop and predictable disturbance off;
CG2 : Lq2 in the loop and predictable disturbance on/off or off/on transition;
CG3 : Lq1 in the loop and predictable disturbance on.

The selection of the CG to be applied is handled by the block ”CG selector” (see fig.5) that makes use of the input T_{dinf}. In order to guaranteed the satisfaction of all the prescribed constraint irrespective of model uncertainties the CG’s have been designed on the same incremental model for the plant (depicted in section IV-A) with the LQ primal controller specified above in the three cases. The only difference has regarded the assumed admissible load disturbance ranges. Specifically,

CG1 : $\mathbf{D}_1 \in [3, 8]$ (Nm) and $\mathbf{D}_2 = \{0\}$ (Nm);
CG2 : $\mathbf{D}_1 \in [3, 8]$ (Nm) and $\mathbf{D}_2 = [0, 12]$ (Nm);
CG3 : $\mathbf{D}_1 \in [3, 8]$ (Nm) and $\mathbf{D}_2 = \{12\}$ (Nm).

For both “on/off” and “off/on” transitions, an upper-bound for $\bar{k}$ was determined in 40 steps (0.4 sec.) and it has been used as a dwell time before switching.

Robustness with respect to disturbance and unmodeled dynamics is achieve via an algorithm for the selection of command to give to primal controller that esteem in preservative way the evolution of disturbance.

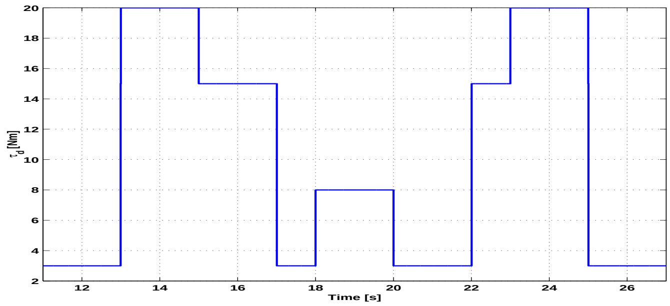

Fig. 6. Load disturbances

C. Simulation

In this section we report some simulation results obtained applying the proposed LQ-CG hybrid control strategy illustrated in Section II to the nonlinear hybrid model of the plant described in Section II. Since the idle speed controller has been designed in order to meet the constraint and robust specifications for the discrete-event model described in Section IV-B, which is a relaxation of the original engine hybrid model given in Section II, then we have the guarantee that the controller is correct by construction.

Furthermore, simulations show that the discrete-time approximation of the plant described in Section III is good enough since the performances of the hybrid closed loop system are satisfactory in terms of fuel consumption.

In Figure 7 simulation results obtained with a set-point $r = [710,\ 2]$ and a load torque disturbance sequence as in Figure 6 are reported. Minimizations of fuel consumption has been obtained essentially via a lower consumption during steady-state phases corresponding to the imposition via the CG action of the lowest possible sustainable idle speed compatible with the actual load.

During the control action, the CG chooses the lower possible set-point in every circumstance. The effect of the CG is especially apparent in the evolution of the engine speed. In fact the engine speed is always at the lowest level – to minimize fuel consumption – compatible with loads, so that the constraints are satisfied for all time. The good performances of the proposed solution are manifested by the injected fuel rate profile, which is never higher than the LQ controller without CG and is strictly lower than it in steady-state.

V. Conclusion

In this paper the idle speed control design problem for an automotive GDI engine has been considered. The problem has been formalized as a robust constrained optimization control problem for a hybrid model of the engine. The proposed solution has been obtained applying the command governor approach, which allows to handle both constraints and robustness. By this approach, fuel consumption due to transients was reduced about of $50\ \%$ with respect to the case of a single optimal LQ controller, essentially for the freedom in tuning the primal LQ switching control structure without the need of taking into account the constraint. The overall consumption reduction was about $2\ \%$.

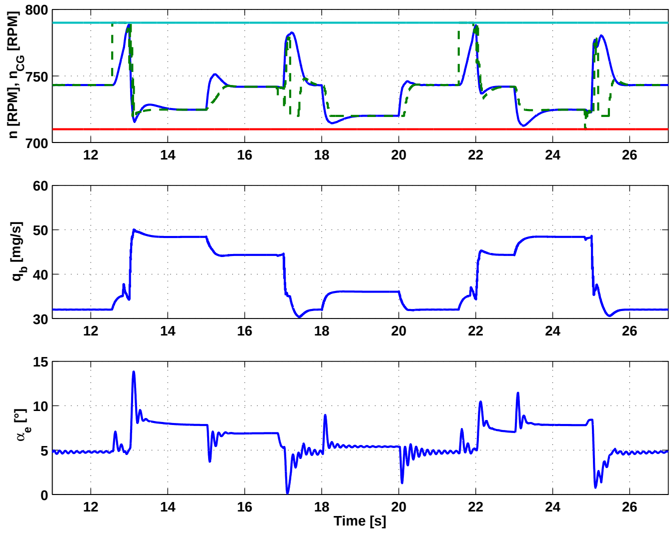

Fig. 7. From top to down: engine speed (solid) and the CG set-point (dashed); the injected fuel rate; the throttle angle.

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